The Graviton Production in a Hot Homogeneous Isotropic Universe

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Abstract

It is shown that the RTG predicts an opportunity of the intensive production of gravitons at the early stage of evolution of the homogeneous isotropic Universe. A hypothesis is suggested that the produced gas of gravitons could be just the "dark matter" which presently manifests itself as a "missing mass" in our Universe.

In publications [1,2] an opportunity of graviton production in the Universe was studied in detail. In paper [2] the following formula for the rate of graviton production in the homogeneous and isotropic Universe was derived

$$\frac{1}{\sqrt{-g}} \, \frac{d}{dt}(\sqrt{-g}\, n_g) = \frac{1}{288\pi} R^2, \tag{1}$$

where it was supposed that

$$\frac{R^2}{R_{\rho\lambda\mu\nu} R^{\rho\lambda\mu\nu}} \ll 1, \tag{2}$$

and R is the scalar curvature, $R_{\rho\lambda\mu\nu}$ is the Riemannian curvature tensor.

The following equation of state takes place in the hot Universe for radiation dominated stage of its evolution

$$p = \frac{1}{3}\rho c^2. \tag{3}$$

But as, according to the General Relativity Theory (GRT), the scalar curvature R is exactly zero at this stage of the Universe evolution, the authors of papers [1,2] came to the conclusion that production of gravitons in the

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hot homogeneous and isotropic Universe does not occur. In publication [1] attention was paid also to the fact that the production of gravitons apparently forbids isotropic singularities, in the vicinity of which the equation of state should be as follows

$$p > \frac{1}{3}\rho c^2. \tag{4}$$

This conclusion has apparently arisen because in this case the scalar curvature R would become arbitrary large, and therefore there should be an extremely intensive production of gravitons, and consequently in presence of a singularity this would result in contradiction to the modern data on the density of matter in the Universe.

In the Relativistic Theory of Gravitation (RTG), which views a gravitational field as a physical one with spins 2 and 0 and propagating in the Minkowski space, completely another situation arises: the evolution of the homogeneous and isotropic Universe is determined by other equations [3,4] and (extremely important) here there are no singularities:

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) - 2\omega\left(1 - \frac{1}{a^6}\right), \tag{5}$$

$$H^2 \equiv \left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\omega}{a^6}\left(1 - \frac{3a^4}{a_{\max}^4} + 2a^6\right), \tag{6}$$

where

$$\omega = \frac{1}{12}\left(\frac{mc^2}{\hbar}\right)^2, \quad m \text{ is the graviton mass.} \tag{7}$$

It follows [3,4] from these equations that for a radiation dominated stage of the Universe evolution in the domain of the small values of the scale factor $a(\tau)$ the following equation takes place:

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{\omega}{a^6}, \quad \text{where } \dot{a} = \frac{da}{d\tau}. \tag{8}$$

In the GRT l.h.s. of Eq. (8) in radiation dominated region is exactly zero, and therefore, Friedman stage takes place, if scale factor $a(\tau)$ varies according to the law $\sqrt{\tau}$. In RTG, according to Eq.(8), there is an "under-Friedman" stage in the radiation dominated phase of evolution of the Universe. The

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scalar curvature R for the homogeneous and isotropic Universe is as follows

$$R = -\frac{6}{c^2}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right].$$ (9)

On the basis of Eq. (8) we have

$$R = -\frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \frac{1}{a^6}.$$ (10)

From equations (6) it follows that the scale factor $a(\tau)$ cannot become zero, and its minimal value is as follows

$$a_{\min} = \left(\frac{\rho_{\min}}{2\rho_{\max}}\right)^{1/6},$$ (11)

where

$$\rho_{\min} = \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2,$$ (12)

and the maximal density of matter in the gravitational field $\rho_{\max}$ is in fact an integral of motion and it is not determined by the theory.

On the basis of Eqs. (10), (11) and (12) it follows that at the moment when the maximal density of matter is reached, the scalar curvature of effective Riemannian space takes the following value

$$R = -\frac{16\pi G}{c^2}\rho_{\max}.$$ (13)

At this instant of time the Hubble "constant" H is precisely zero. We can see from formula (13) that in the RTG, just opposite to the GRT, the scalar curvature R in radiation dominated stage of the Universe evolution is not zero. Moreover, it can be large enough as it is determined by the peak density of matter $\rho_{\max}$ in the gravitational field.

Thus, according to RTG, in the radiation dominated phase of the Universe evolution there is an "under-Friedman" stage, in which the scalar curvature R is not only different from zero, but also can be large enough, as it is determined by the peak density of matter $\rho_{\max}$. To determine the rate of graviton production we cannot take advantage of formula (1), as it is obtained in approximation (2), which in our case is not fulfilled.

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If on the base of dimensional reasons we assume that the rate of graviton production in general depends only on the following quantities

$$R^2, \ R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu}, \tag{14}$$

than it is necessary to pick such a time interval, during which the Hubble "constant" reaches the maximum, as after that there soon occurs the Friedman stage. From Eqs. (5) and (6) it is easy to find, that H reaches its maximum at the instant of time when the scale factor $a(\tau)$ is as follows

$$a^2(\tau) = \frac{3}{2}a_{\min}^2. \tag{15}$$

By using Eq. (15) from Eqs. (6) it is discovered that the peak value of the Hubble "constant" is as follows

$$H_{\max} = 3^{-2}(32\pi G\rho_{\max})^{1/2}. \tag{16}$$

At the instant of time when H reaches the maximum the scalar curvature R is as follows

$$R = -\left(\frac{2}{3}\right)^3 16\pi G\frac{\rho_{\max}}{c^2}, \tag{17}$$

$\frac{\ddot{a}}{a}$ is determined by expression

$$\left(\frac{\ddot{a}}{a}\right) = 3^{-4} \cdot 32\pi G\rho_{\max}. \tag{18}$$

The invariant obtained by convolution of a curvature tensor determined from metric of the homogeneous and isotropic Riemannian space is as follows

$$R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu} = \frac{12}{c^4}\left[\left(\frac{\ddot{a}}{a}\right)^2 + \left(\frac{\dot{a}}{a}\right)^4\right]. \tag{19}$$

By substituting (16) and (18) into this expression we obtain

$$R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu} = 8 \cdot 3^{-7}\left(\frac{32\pi G}{c^2}\rho_{\max}\right)^2. \tag{20}$$

It is necessary to mention that the Hubble "constant" varies from zero value to the peak value $H_{\max}$ determined by formula (16) for a rather small time interval given by the following expression [3,4]:

$$\tau = \left(\frac{3}{32\pi G\rho_{\max}}\right)^{1/2} \cdot \left[\frac{\sqrt{3}}{2} + \ln\left(\sqrt{\frac{3}{2}} + \sqrt{\frac{1}{2}}\right)\right]. \tag{21}$$

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If the rate of graviton production is determined by quantities (14) there can be created enough large number of gravitons for a time interval (21), if density $\rho_{\max}$ will have a significant value. But if it will be much less than the Planck density then produced gravitons at once become free, and their energy further will diminish due to the red shift.

Thus, there should arise a relativistic relict gravitational background of a non-thermal origin. The gravitons interact among themselves strongly enough as their coupling constant is equal to unity. This circumstance can break homogeneity of a relict gravitational background of a non-thermal origin at sufficient density of gravitons. From dimensional reasons the general number of produced quanta of the gravitational field in cubic centimeter of volume will be proportional to the following quantities

$$cR^2\tau, \quad c(R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu})\tau, \tag{22}$$

where $R^2, R_{\rho\lambda\mu\nu}R^{\rho\lambda\mu\nu}, \tau$ are given by expressions (17), (20) and (21). It follows from these formulas that the rate of graviton production in the hot radiation dominated phase of evolution of the Universe is mainly determined by the peak density of matter $\rho_{\max}$. Knowing more detailed pattern of graviton production it would be possible to spot the peak value of density of matter, which the Universe had in the present cycle of "expansion". On the other hand, it is possible to state a hypothesis: the gravitational background of a non-thermal origin could be just that "dark matter", which manifest itself as "missing mass" in our Universe . But all this requires more careful analysis.

References

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